TURBULENT FLOW AND HEAT TRANSFER IN A ROTATING SLIT-SHAPED CHANNEL

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A two-parameter turbulence model is used to numerically study flow and heat transfer in a channel rotating about its transverse axis.

It is known that the rotation of a channel about its transverse axis leads to emergence of the flow core and the formation of thin Ekman boundary layers on the surfaces bounding the region of flow in the direction of the rotation axis. This is seen in "pure" form in a channel formed by two parallel planes which are perpendicular to the rotation axis. Laminar flow and heat transfer in such a channel were studied in [1] by analytic methods. In turbulent flow, restructuring of the velocity field involves significant changes in the characteristics of turbulent transport. As shown by experimental data on friction [2, 3], rotation at a certain rate also leads to laminarization of the initial turbulent flow. Below we examine these problems by a numerical method on the basis of a two-parameter turbulence model k–ε [4].

We will examine the flow of an incompressible fluid along a prismatic slit-shaped channel with long parallel sides separated by a distance 2h. The channel rotates with a constant angular velocity ω about the axis which is perpendicular to the sides of the channel. The short end walls determine the general direction of transfer the fluid mass.

We will introduce a Cartesian coordinate system rigidly connected with the channel and oriented so that the y axis is directed along the rotation axis and the z axis is parallel to the channel walls in the flow direction. The coordinate origin is located in the middle plane. Assuming averaged flow which is steady-state over time and which is developed with respect to the longitudinal coordinate z, we will ignore end effects at the end walls. Thus, the fields of velocity and the turbulence characteristics are assumed to be independent only of the transverse coordinate y.

We will study heat transfer on the assumption of constancy of the heat flux along the walls, development with respect to z, and uniformity with respect to the transverse coordinate x. We will ignore the effect of buoyancy and dissipative processes caused by deformation of the mean velocity field.

Introducing the modified pressure $p^* = \langle p \rangle/\rho - \omega^2 r^2/2$, we write the following system of Reynolds equations and energy equations with allowance for the above assumptions:

$$\frac{d}{dy}\left(\nu \frac{dU}{dy} - \langle uv \rangle\right) = \frac{\partial p^*}{\partial x} + 2\omega W, \tag{1}$$

$$\frac{d}{dy}\left(\nu \frac{dW}{dy} - \langle wv \rangle\right) = \frac{\partial p^*}{\partial z} - 2\omega U, \tag{2}$$

$$\frac{\partial}{\partial y}\left(\frac{\nu}{\mathrm{Pr}} \frac{\partial T}{\partial y} - \langle tv \rangle\right) = W \frac{\partial T}{\partial z}. \tag{3}$$

We establish the boundary conditions:

$$U = W = 0,\ T = T_w = \sigma z + \mathrm{const} \quad \text{at} \quad y = \pm h.$$

It follows from (1), (2) that p* is a linear function of the coordinates x and z:

$$p^* = Ax + Bz + \mathrm{const}.$$

To determine the constants A and B, we have two integral equations

$$\int_{-h}^{h} W dy = 2hW_m, \quad \int_{-h}^{h} U dy = 0,$$

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the first of which is the condition of flow at a specified rate; the second is a consequence of the impermeability of the end walls of the channel. To close the problem we adopt the hypothesis

$$-\langle uv \rangle = \nu_t \frac{dU}{dy}, \quad -\langle wv \rangle = \nu_t \frac{dW}{dy}, \quad -\langle tv \rangle = \nu_t \frac{dT}{dy} \tag{4}$$

and use the k—ε turbulence model [4]. The latter is valid for calculating flows with low values of the turbulent Reynolds number.

In the flow being studied, the vector of the Coriolis body force lies in a plane perpendicular to the direction of the shift in velocity. Analysis of the differential equations for the Reynolds stresses shows that in this case there are no grounds for expecting significant effects from the direct action of rotation on turbulent transport. In any case, the errors caused by omitting the effects of direct action are of the same order of magnitude as those due to assumption (4) on the scalar character of ν_t. In this sense, the motion being studied is similar to the flow at the surface of a rotating disk. The k—ε model can be used very successfully in calculations of the flow on a rotating disk [5]. These facts and the conclusions reached in [6] regarding the values of the empirical constants dictated the use of the model in [4] in the unchanged form:

$$\frac{d}{dy}\left[(\nu+\nu_t)\frac{dk}{dy}\right] = -G + \varepsilon + 2\nu\left(\frac{dk^{1/2}}{dy}\right)^2,$$

$$\frac{d}{dy}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{d\varepsilon}{dy}\right] = -c_1\frac{\varepsilon G}{k} + c_2 f_\mu \frac{\varepsilon^2}{k} - c_3 G_\mu,$$

$$G = \nu_t\left[\left(\frac{dU}{dy}\right)^2 + \left(\frac{dW}{dy}\right)^2\right], \quad G_\mu = \nu\nu_t\left\{\frac{d}{dy}\left[\left(\frac{dU}{dy}\right)^2 + \left(\frac{dW}{dy}\right)^2\right]^{\frac{1}{2}}\right\}^2,$$

$$\nu_t = c_4 \exp\left[-2.5/(1+\mathrm{Re}_t/50)\right]\frac{k^2}{\varepsilon},$$

$$f_\mu = 1.0 - 0.3\exp(-\mathrm{Re}_t^2), \quad \mathrm{Re}_t = \frac{k^2}{\nu\varepsilon},$$

$$c_1 = 1.55;\ c_2 = 2.0;\ c_3 = 2.0;\ c_4 = 0.09;\ \sigma_\varepsilon = 1.3,$$

$$k = 0,\ \varepsilon = 0 \quad \text{at} \quad y = \pm h.$$

Assuming

$$T = T_w + \sigma h \Theta(y),$$

we write the following equation from (3) and (4) for the temperature profile:

$$\frac{d}{dy}\left[\left(\frac{\nu}{\mathrm{Pr}} + \nu_t\right)\frac{d\Theta}{dy}\right] = \frac{W}{h}.$$

Using the equation $\mathrm{Nu} = 2hq_w/\lambda_*(T_w - T_m)$, we obtain the following from the heat balance equation:

$$\mathrm{Nu} = -2\mathrm{PrRe}W_m h / \int_{-h}^{h} W\Theta dy.$$

As the determining criteria of the flow we select the Reynolds number $\mathrm{Re} = 2W_m h/\nu$ and the complex $\gamma = \omega h^2/\nu$. When presenting the results of solution of the stated problem, we will have in mind the symmetry of the flow relative to the plane y = 0.

The system of differential equations was integrated for U, W, k, ε, and Θ by a numerical method, with approximation of the derivatives by finite differences in accordance with a conservative scheme on a nonuniform grid [7]. The nodes were moved closer together at the wall in accordance with the following law of geometric progression:

$$\xi_m = \xi_1 \frac{1-q^m}{1-q}, \quad \xi_1 = \frac{1-q}{1-q^{M-1}}, \quad m = \overline{2, M-1},$$

where $\xi = 1 - y/h$, M is the number of grid nodes in the range $0 \leqslant \xi \leqslant 1$.

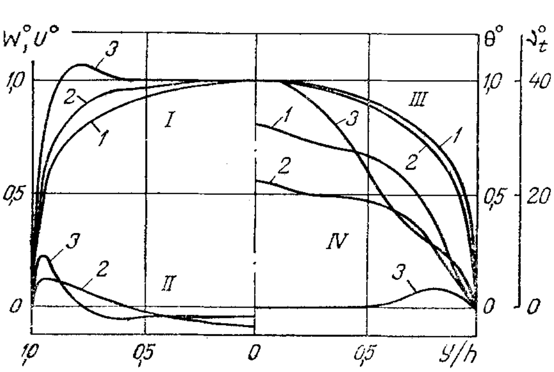

Fig. 1

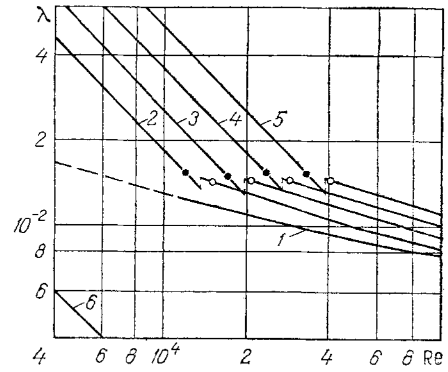

Fig. 2

Fig. 1. Profiles of the components of velocity, temperature, and eddy viscosity at Re = 12,400; Pr = 0.71: 1) $\gamma = 0$; 2) 100; 3) 300; I) W°; II) U°; III) θ°; IV) ν.

Fig. 2. Friction coefficient: 1) $\gamma = 0$; 2) 500; 3) 1000; 4) 2000; 5) 4000; 6) laminar flow in a stationary channel.

We used the establishment method to find the steady-state solution. The time step τ_m was changed from node to node in proportion to the local step with respect to the coordinate $\tau_m = \alpha(\xi_{m+1} - \xi_m)$ but it remained unchanged during the establishment process.

The method used to linearize the equations for k and ε to a significant degree determines the permissible time step — more accurately, it determines the multiplier α. Calculations showed the expediency of using the following form of linearization:

$$\frac{k_n - k_{n+1}}{\tau_m} + \frac{d}{dy}\left[(\nu + \nu_t)_n \frac{dk_{n+1}}{dy}\right] + G_n - \frac{\varepsilon_n}{k_n} k_{n+1} = 2\nu\left(\frac{dk_n^{1/2}}{dy}\right)^2,$$

$$\frac{\varepsilon_n - \varepsilon_{n+1}}{\tau_m} + \frac{d}{dy}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)_n \frac{d\varepsilon_{n+1}}{dy}\right] + c_1 \frac{\varepsilon_n G_n}{k_n} - c_2 f_\mu \frac{\varepsilon_n}{k_n}\varepsilon_{n+1} = -c_3 (G_\mu)_n.$$

Systems of algebraic equations for $k_{n+1,m}$ and $\varepsilon_{n+1,m}$ were solved by the trial-run method at each new (n + 1) time step for the scalar equations, while systems for $U_{n+1,m}$ and $W_{n+1,m}$ were solved simultaneously by the same method for the three-point vector equations.

We used the Prandtl hypothesis to construct the initial distribution $W_0(y)$. Let the agitation l be specified from the Prandtl—Nikuradze formula with allowance for the Van Driest modification [8]. We assumed that $U_0(y) = 0$. The initial distributions for k_0 and ε_0 were determined from the Townsend formula:

$$k_0(y) = -c_0 \langle wv \rangle = c_0 l^2 \left(\frac{dW}{dy}\right)^2,$$

with $c_0 = 10/3$ and the assumption of locally equilibrium turbulence $\varepsilon_0(y) = G_0(y)$. The constructed initial distributions approximately described the fields of W, k, and ε in turbulent flow along a stationary plane-parallel channel.

We used the steady-state distributions W(y) and $\nu_t(y)$ in solving the linear equation for the temperature profile. The main calculations were performed with M = 51 (q = 1.1) and M = 101 (q = 1.04). As a test we choose a problem on laminar flow in a slit-shaped channel for which an analytic solution is known [1]. We also choose results of calculations of turbulent flow in a stationary channel [4, 9] performed using the same turbulence model. In all cases, we obtained nearly the same results.

Figure 1 shows the effect of rotation on the fields of mean velocity, temperature, and eddy viscosity. Secondary flow develops with an increase in the rotation rate, the profile W(y) becomes fuller, and a core emerges in the flow with a uniform velocity distribution along lines parallel to the rotation axis and shear boundary layers which can be classified as Ekman layers.

It follows from analysis of the fields of k that an increase in the parameter γ is accompanied by gradual degeneration of pulsative motion in the flow core. It is important to

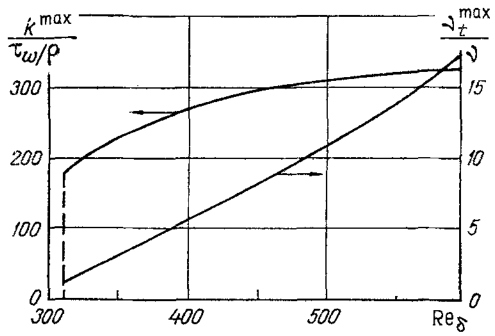

Fig. 3

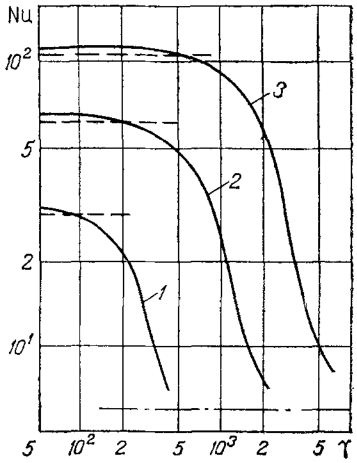

Fig. 4

Fig. 3. Dependence of the maximum values of kinetic turbulence energy and eddy viscosity in the Ekman layer on the criterion Re_δ.

Fig. 4. Dependence of the rate of heat transfer on the rotation parameter: 1) Re = 12,400; 2) 30,400; 3) 61,600; the dashed lines correspond to $\gamma = 0$; the dot—dash line corresponds to Nu at $\gamma \to \infty$ for the laminar regime [1].

emphasize here that the maximum value of the kinetic turbulence energy expended in the transition region from the viscous sublayer to the turbulence zone changes relatively little. Only when a certain rotation rate is reached (at $\gamma \gtrsim 400$ for Re = 12,400) is there a critical reduction in k to zero throughout the flow. Thus, with sufficiently large values of γ, the pulsative motion initially introduced into the flow decays complete during the establishment process.

The level of ν_t changes sharply with an increase in γ. This is connected mainly with a decrease in the turbulence scale caused by thinning of the shear layer. The changes in the field $\nu_t(y)$ also involve a fundamental restructuring of the temperature profile.

The results of a large number of calculated variants were used to determine the dependence of the friction coefficient of the rotating channel

$$\lambda = -\frac{4h}{W_m^2}\frac{\partial p^*}{\partial z}$$

on the criteria Re and γ (Fig. 2). The dashed extension of curve 1 was constructed from the data in [4]. The straight line 6 corresponds to the friction law in a plane-parallel nonrotating channel with laminar motion ($\lambda = 24/Re$). The branches of curves 2-5 for laminar and turbulent regimes are tentatively connected by dashed segments.

The points in Fig. 2 show the boundaries of the scatter of the data in [2] with respect to the values of Re, which were determined in an analysis of empirical relations for the friction coefficient of a channel of square cross section. The dark points denote the lower boundary, while the clear points denote the upper boundary.

At Re $\gtrsim$ 5000, laminarization of the flow occurs at the same values of γ for which there is clear emergence of the flow core and boundary layers. This makes it necessary to change over to the criterion $Re_\delta = W_c/\sqrt{\omega\nu}$, which does not contain the scale h and which represents the Reynolds number plotted from the characteristic thickness $\delta = \sqrt{\nu/\omega}$ of the laminar Ekman layer.

Figure 3 shows the dependence of two characteristics of the rate of turbulent transport on Re_δ. It is apparent that as Re_δ approaches 300, eddy viscosity monotonically decreases to a very low value ($\nu_t^{max} \simeq \nu$). We did not obtain any solutions with nontrivial values of k or ν_t at $Re_\delta < 310$. In the range $310 < Re_\delta \lesssim 450$, either a solution with $\nu_t \neq 0$ or a completely laminar regime may be obtained, depending on the factors α, M, and q (with specified values of M and q, an increase in α leads to decay of k and ν_t at all higher values of Re_δ).

It should be noted that the authors of [10], by analyzing spectra of velocity pulsations which they measured in a transitional Ekman layer, concluded that turbulence ends at $Re_\delta \simeq 300$. Thus, well-known empirical data confirms the results of the theoretical model used here.

Figure 4 shows the relation $Nu(\gamma)$ at $Pr = 0.71$ and three values of Re. The values of Nu obtained for $\gamma = 0$ are roughly 15% lower than those found from the empirical relation

$$Nu = 0.012\,(2Re)^{0.8}Pr^{0.43},$$

which was proposed in [11] for heat transfer in a stationary plane-parallel channel far from the inlet. When rotation is superimposed, the rate of heat transfer initially increases somewhat and then decreases sharply and approaches the level corresponding to the laminar regime [1]. Such behavior of $Nu(\gamma)$ at Re = const is due to "competition" between two factors which affect heat transfer. The first is connected with filling out of the profile $W(y)$. This, as in the case of the laminar regime [1], leads to an increase in Nu. The second effect consists of a decrease in turbulent heat transfer in the direction of the y axis. For the chosen model [4], this is directly connected with the decrease in eddy viscosity ν_t. The data in Fig. 4 shows that the second effect becomes dominant at $\gamma/Re \gtrsim 0.01$.

NOTATION

h, half of the channel height; ω, angular velocity; x, y, z, rotating Cartesian coordinate system; U, V, W, u, v, w, mean and pulsative components of velocity and directions of the x, y, and z axes; T, mean temperature; t, temperature pulsation; p, pressure; ρ, density; p^*, modified pressure; r, shortest distance to rotation axis; k, ε, kinetic turbulence energy and its rate of dissipation; ν, ν_t, kinematic molecular and eddy viscosities; Θ, dimensionless temperature; Re, Nu, Pr, Reynolds, Nusselt, and Prandtl numbers; γ, rotation parameter; Re_δ, Reynolds number for the Ekman layer; λ, friction coefficient; λ_*, thermal conductivity; T_w, q_w, τ_w, temperature, heat flux, and total shear stress on the wall; W_m, flow-rate-mean velocity; T_m, mass-mean temperature; W_c, value of the component of W in the middle plane; $U^\circ = U/W_c$, $W^\circ = W/W_c$, $\Theta^\circ = \Theta/\Theta(0)$, $\nu_t^\circ = \nu_t/\nu$, dimensionless quantities; $\langle\ \rangle$, symbol denoting averaging over time; A, B, σ, constants of integration.

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